24. $\lim_{x \to 0^+} (\sin x)^x$ $\quad 0^0$

25. $\lim_{x \to 0^+} \left(1 + \frac{1}{x}\right)^x$ $\quad \infty^0$

Jan 23-7:38 AM

8.3 Relative Rates of Growth

Definitions: Faster, Slower, Same-rate Growth

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \infty$ $\quad$ f : faster

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$ $\quad$ g : faster

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = L$ $\quad$ f , g : same

↑ finite, not 0

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$$\lim_{x\to\infty} \frac{10x}{x} = 10$$ $10x$, x same rate (order of mag.)

$$\lim_{x\to\infty} \frac{x^2}{x} = \infty$$ x^2 faster

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Which function grows faster?

e^x, x^2

$$\lim_{x\to\infty} \frac{e^x}{x^2} \quad \frac{\infty}{\infty} \qquad \lim_{x\to\infty} \frac{e^x}{2x} = \lim_{x\to\infty} \frac{e^x}{2} = \infty$$

e^x faster than x^2

$\ln x, x, x^2$

x^2 fastest

x

$\ln x$

$$\lim_{x\to\infty} \frac{x^2}{x} = \infty$$

$$\lim_{x\to\infty} \frac{x}{\ln x} = \lim_{x\to\infty} \frac{1}{1/x} = x$$

$\frac{\infty}{\infty}$ $\lim_{x\to\infty} x = \infty$

x faster than $\ln x$

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Show the functions grow at the same rate

$x, x+\sin x$

$$\lim_{x\to\infty} \frac{x}{x+\sin x} = \lim_{x\to\infty} \frac{1}{1+\cos x}$$

$$\lim_{x\to\infty} \frac{x+\sin x}{x} = \lim_{x\to\infty} 1 + \overset{0}{\frac{\sin x}{x}} = 1$$

$\log_a x, \log_b x$

$$\lim_{x\to\infty} \frac{\log_a x}{\log_b x} = \lim_{x\to\infty} \frac{\frac{1}{x \ln a}}{\frac{1}{x \ln b}} = \frac{\cancel{x}\ln b}{\cancel{x}\ln a}$$

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Transitivity of Growing Rates

if f same rate as g
and g same rate as h then f same as h

Show the functions grow at the same rate by comparing both with x

$\sqrt{x^2+5}, (2\sqrt{x}-1)^2$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{x} = \lim_{x\to\infty} \frac{\frac{1}{2}(x^2+5)^{-\frac{1}{2}}2x}{1} = \frac{x}{\sqrt{x^2+5}}$$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{\sqrt{x^2}} = \lim_{x\to\infty} \sqrt{\frac{x^2+5}{x^2}} = \lim_{x\to\infty} \sqrt{1+\frac{5}{x^2}}$$
$$= 1$$

same

$$\lim_{x\to\infty} \frac{(2\sqrt{x}-1)^2}{(\sqrt{x})^2} = \lim_{x\to\infty} \left(\frac{2\sqrt{x}-1}{\sqrt{x}}\right)^2$$
$$= \lim_{x\to\infty} \left(\frac{2\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}}\right)^2$$
$$= \lim_{x\to\infty} \left(2 - \frac{1}{\sqrt{x}}\right)^2 = 4$$

same

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Sequential vs Binary Searches

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